

Three Fish Limit

Some friends are camping near the shore. One friend suggests that they go fishing. Six boats are available at the campsite. Each boat holds two people and the friends are excited because that is exactly how many they need. The rule on this lake is that each person can catch only three fish. How many fish did the friends catch that day if each friend caught the limit? Show all your mathematical thinking.

Three Fish Limit

Common Core Task Alignments

Mathematical Practices: 1, 3, 4, 5, 6, 7

Grade 3 Content Standards:
3.OA.A.3

Task

Some friends are camping near the shore. One friend suggests that they go fishing. Six boats are available at the campsite. Each boat holds two people and the friends are excited because that is exactly how many they need. The rule on this lake is that each person can catch only three fish. How many fish did the friends catch that day if each friend caught the limit? Show all your mathematical thinking.

Common Core Content Standards and Evidence

3.OA Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Exemplars Task-Specific Evidence

This task requires students to use repeated addition or multiplication to find the number of people who are camping and the number of fish they catch.

Underlying Mathematical Concepts

- 2 to 1
- 3 to 1
- Addition/Counting on
- Multiplication within 100
- Number sense to 36
- Combining

Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Table
- Tally chart
- Number line

Possible Mathematical Vocabulary/Symbolic Representation

- Model
- Diagram/Key
- Table
- Tally chart
- Number line
- Pattern
- Multiples
- Per
- Odd/Even
- More/Less than
- 1/2
- Rule
- $2 \cdot b = p$ ($b + b = p$), $6 \cdot b = f$
- Variables
- Total
- Product
- Dozen
- Pair
- Input/Output
- Array
- Graph
- Axis
- Sets
- Groups

Possible Solutions

The friends caught 36 fish.

Boat	People	Fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

Rule

b is boat
p is people
f is fish

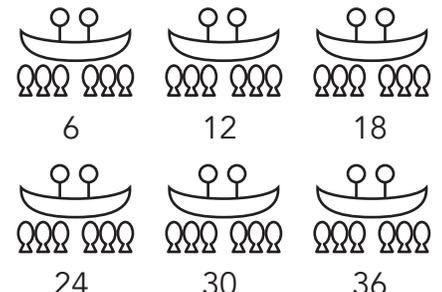
$2 \cdot b = p$
 $3 \cdot p = f$
 $6 \cdot b = f$

Key

 is 1 Person

 is 1 Fish

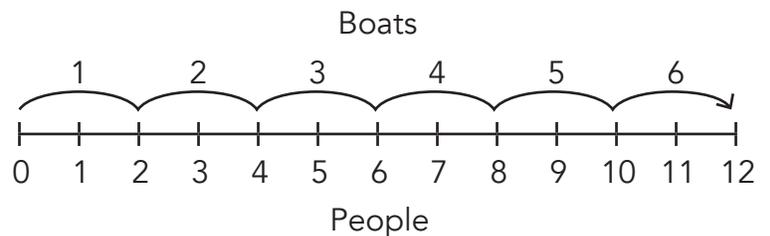
 is 1 Boat



$2 \times 6 = 12$
 $3 \times 12 = 36$
 $6 \times 6 = 36$

$6 + 6 + 6 + 6 + 6 + 6 = 36$
 $6 \times 6 = 36$

Boat	1	2	3	4	5	6
People						
Fish						



Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- Patterns: Boat +1, People +2, Fish +6.
- The people and amount of fish caught are even numbers.
- 1 person = $\frac{1}{2}$ boat.
- 2 people is a pair.
- Multiples of 2 and 6 are even numbers.
- There are 4 more fish than people.
- Graph input/outputs on graph paper.
- Rules: $2 \cdot b = p$, $6 \cdot b = f$ (b is boat, p is person, f is fish)
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.

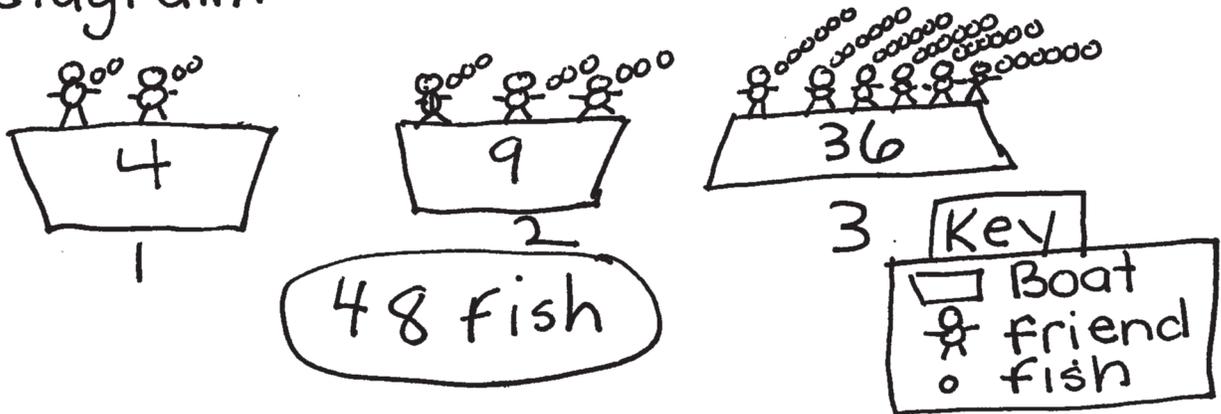
Novice Scoring Rationales

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Novice</i>	The student's strategy of using a diagram to show three boats with two friends in a boat catching two fish each, three friends in a boat catching three fish and six friends in a boat catching six fish would not work to solve this task. The student's answer, "48 fish," is not correct.
Reasoning Proof <i>Novice</i>	The student does not show any correct conceptual understanding of the problem. It appears that the student confuses three fish with three boats and then has the friends catch a number of fish equal to the number of friends in each boat.
Communication <i>Practitioner</i>	The student correctly uses the mathematical terms <i>diagram</i> and <i>key</i> . Although the diagram represents incorrect data, the use of the communication terms is correct.
Connections <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
Representation <i>Apprentice</i>	The use of a diagram is appropriate to this task but the student's work is not correct. The student does not represent six boats with two friends in each boat and each friend catching three fish.

Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	P	N	A	N

I have to find fish. I will make a diagram.



Boat 3 caught the most fish.
The fish pattern is 2 fish, 3 fish,
6 fish.

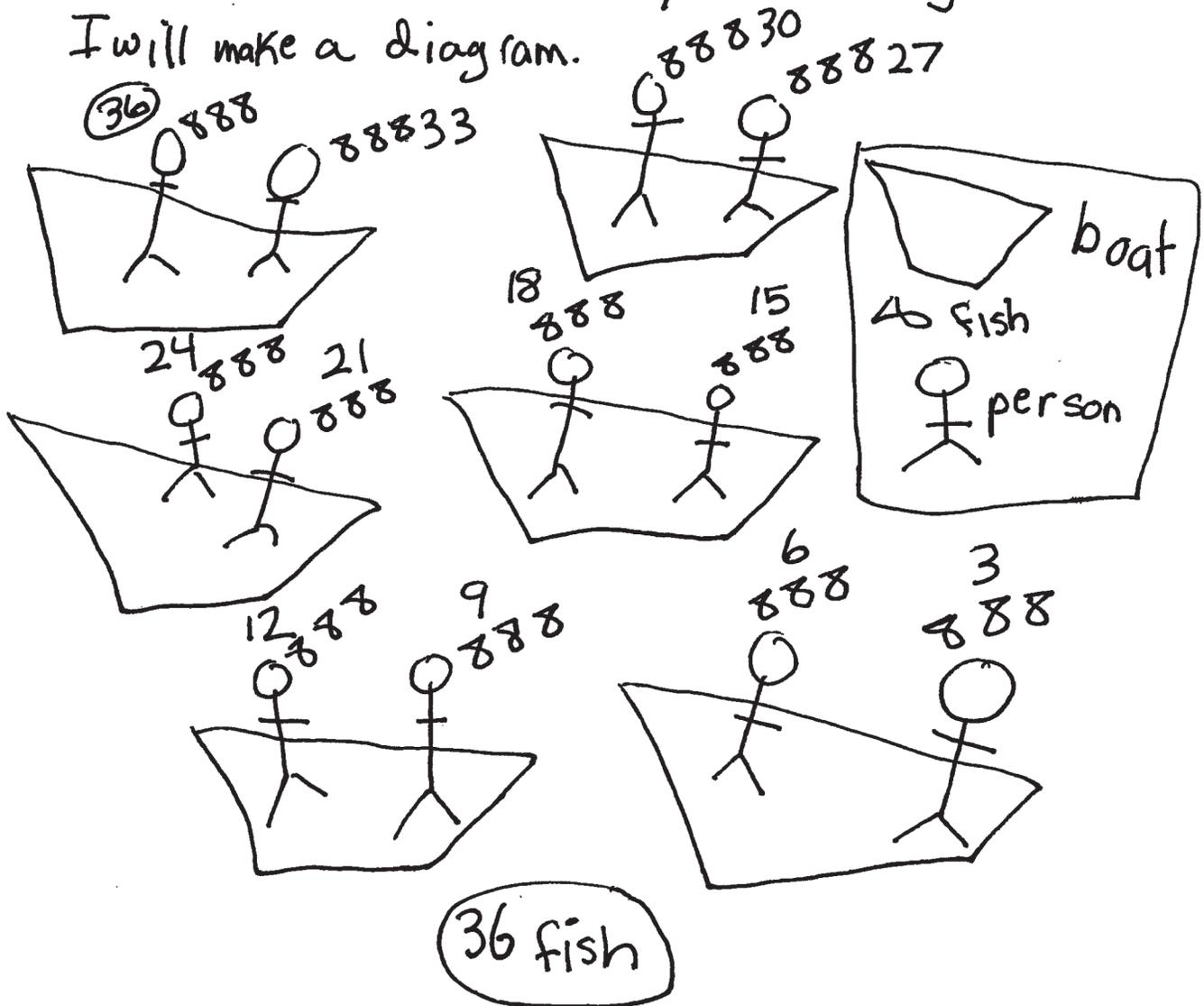
Apprentice Scoring Rationales, Student 1

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Practitioner</i>	The student's strategy of using a diagram and a key to represent the six boats, two friends per boat, and three fish per friend would work to solve this task. The student states a correct answer, "36 fish."
Reasoning Proof <i>Practitioner</i>	The student's arguments are constructed with adequate mathematical basis. The student's solution is systematic and supports correct reasoning.
Communication <i>Apprentice</i>	The student correctly uses the mathematical term <i>diagram</i> .
Connections <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
Representation <i>Practitioner</i>	The student's diagram is appropriate and accurate. The student provides labels in a key for a boat, fish, and person. The student does not have to use the term key, but it would have provided another mathematical term to the student's solution.

Apprentice, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	A	N	P	A

I need to find out how many fish caught.
I will make a diagram.



Apprentice Scoring Rationales, Student 2

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Apprentice</i>	The student's strategy of making a table for boats and fish would work to solve the task but the fish column only represents the fish caught by one friend per boat and not two friends per boat. The student's answer, "The answer is 18 fish," is incorrect.
Reasoning Proof <i>Apprentice</i>	The student has a flaw in her/his reasoning. The running total of fish in the second column in the student's table only represents three fish per friend and not six fish for two friends.
Communication <i>Apprentice</i>	The student correctly uses the mathematical term <i>table</i> .
Connections <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
Representation <i>Apprentice</i>	The student's table is appropriate but not accurate. The fish column has incorrect data. The data should represent 6, 12, 18, 24, 30, 36 fish.

Apprentice, Student 2

P/S	R/P	Com	Con	Rep	A/Level
A	A	A	N	A	A

I have to find how many fish they caught on the lake to solve the problem. I will use a table.

boats	Fish
1	3
2	6
3	9
4	12
5	15
6	18

The answer is
18 Fish

I figured my answer on my table. There are 18 fish.

Practitioner Scoring Rationales, Student 1

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Practitioner</i>	The student's strategy of making a table to show six boats, two people, and a running total of fish caught would work to solve the task. The student states a correct answer, "They caught a total of 36 fishes."
Reasoning Proof <i>Practitioner</i>	The student provides adequate evidence of her/his reasoning both in her/his table and text. The student applies understanding of multiplication by applying the counting by one, two, and six patterns.
Communication <i>Practitioner</i>	The student correctly uses the mathematical terms <i>table</i> , <i>total</i> and <i>pattern</i> .
Connections <i>Practitioner</i>	The student makes the mathematically relevant observations, "The patterns are boats +1, friends + 2, fish caught +6."
Representation <i>Practitioner</i>	The student's table is appropriate and accurate. All necessary labels are provided and the data is correct.

Practitioner, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I will find out how many fish they catch altogether. I will use a table to solve this Problem

boats	friends	fish caught
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

They caught a total of 36 fishes

The patterns are
boat + 1 friend + 2
fish caught + 6

Practitioner Scoring Rationales, Student 2

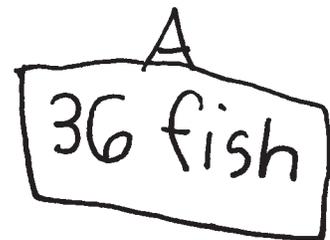
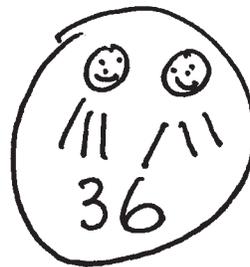
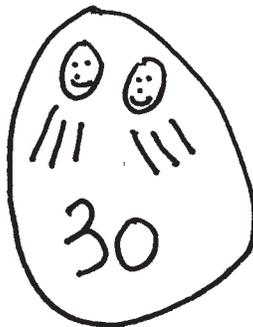
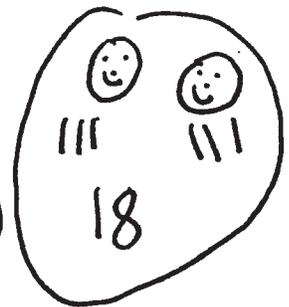
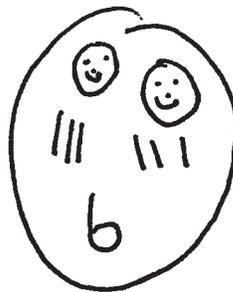
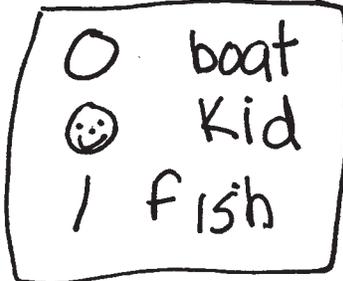
Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Practitioner</i>	The student's strategy of diagramming the boats, kids, and fish, and providing an accurate key works to solve the task. The student states a correct answer, "36 fish."
Reasoning Proof <i>Practitioner</i>	The student demonstrates understanding of the mathematical concept of applying multiplication and patterns to find the total number of fish that can be caught while fishing. The student also applies multiplication and addition in determining the total cost of the 36 fish sold at 50 cents per fish.
Communication <i>Practitioner</i>	The student correctly uses the mathematical terms <i>diagram</i> , <i>key</i> , <i>pattern</i> , <i>dozen</i> and <i>cost</i> . The student correctly uses the mathematical notation \$.50, \$3.00, \$9.00, \$18.00.
Connections <i>Practitioner</i>	The student makes the mathematically relevant observations, "I saw the add 6 pattern for fish," and, "I saw that there were 12 kids and that is a dozen." The student adds her/his own part two of the task by determining that if each fish sold for \$.50, the total cost would be \$18.00.
Representation <i>Practitioner</i>	The student's diagram is appropriate and accurate. A key is used to represent the boats, kids and fish.

Practitioner, Student 2

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I need to find out how many fish.
I will make a diagram.

Key



I saw the add 6 pattern
I saw that there were 12 Kids
and that is a dozen

Practitioner, Student 2 (cont.)

If they sold the fish for \$.50

$$\begin{array}{l} \$.50 \times 6 = \$3.00 \\ \$.50 \times 6 = \$3.00 \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \$9.00 \\ \\ \\ \\ \\ \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \$18.00$$

It would cost \$18.00

Practitioner Scoring Rationales, Student 3

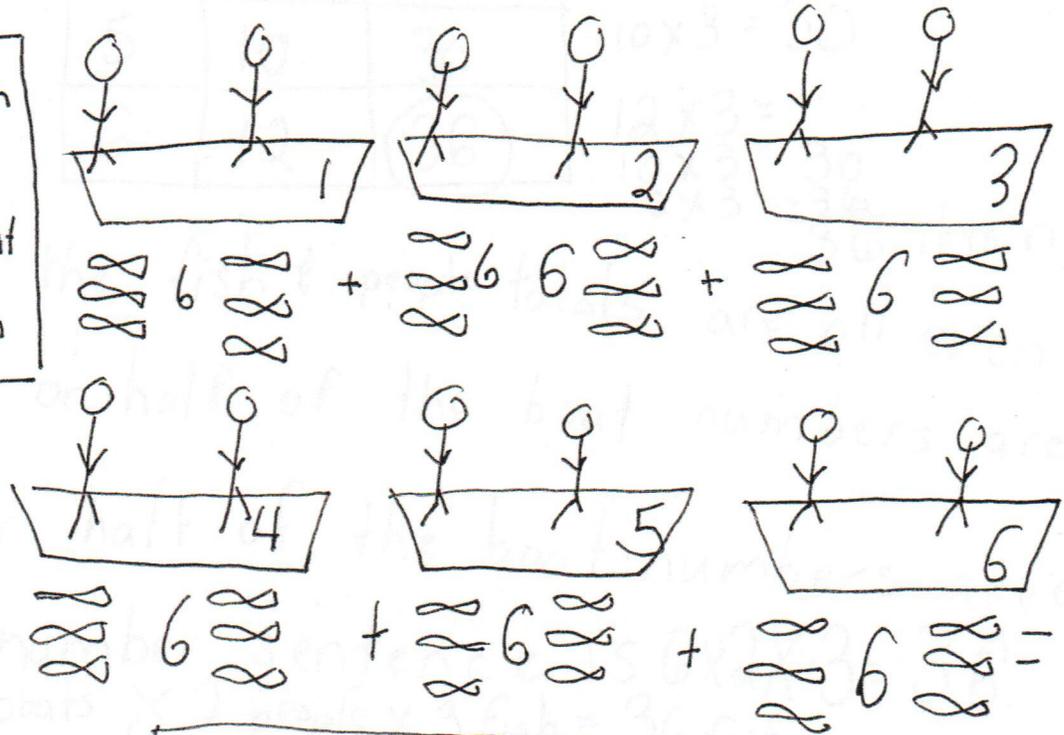
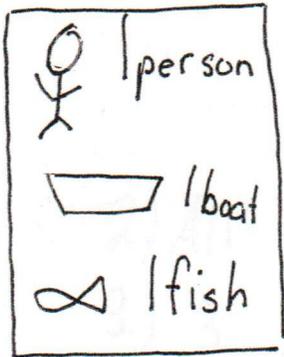
Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Expert</i>	The student's strategy of diagramming the persons, boats, and fish, and providing an accurate key works to solve the task. The student states a correct answer, "36 fish in all." The student verifies her/his answer by using a table to compare with the data in her/his diagram.
Reasoning Proof <i>Expert</i>	The student demonstrates understanding of the mathematical concept of applying multiplication patterns to find the total number of fish that can be caught while fishing. The student also applies multiplication and addition in a table to verify her/his answer. The student brings the understanding of fractions to the task.
Communication <i>Practitioner</i>	The student correctly uses the mathematical terms <i>total, diagram, key, table, even, odd, half, number sentence, dozen, sum</i> . The student correctly uses the mathematical notation $\frac{3}{6}$. All of these terms are considered typical to the Grade three Common Core standards and do not earn a performance level of Expert.
Connections <i>Expert</i>	The student makes the mathematically relevant Practitioner connections, "All the fish & people totals are all even," and, "A number sentence is $6 \times 2 \times 3 = 36$. 6 boats \times 2 people \times 3 fish = 36 fish." The student makes the Expert connection of applying an understanding of fractions to the task. The student states, " $\frac{3}{6}$ or half of the boat numbers are odd," " $\frac{3}{6}$ or half of the boat numbers are even," and "The boat sum is half of the people sum." The student verifies her/his answer by comparing the data on her/his diagram with the data on a table and stating, "it is right."
Representation <i>Expert</i>	The student's diagram is appropriate and accurate. A key is used to represent the persons, boats, and fish. The student's table is appropriate and accurate. All labels are provided and the the entered data is correct. The student compares the data of her/his diagram and table to help verify that the answer is correct.

Practitioner, Student 3

P/S	R/P	Com	Con	Rep	A/Level
E	E	P	E	E	P

I need to find out how many fish they caught total. I will make a diagram.

KEY



Answer: 36 fish in all.

Practitioner, Student 3 (cont.)

Connections

1.) I will prove my answer another way using a table.

total boat	total people	total fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

$$2 \times 3 = 6$$

$$4 \times 3 = 12$$

$$6 \times 3 = 18$$

$$8 \times 3 = 24$$

$$10 \times 3 = 30$$

$$12 \times 3 = 36$$

$$10 \times 3 = 30$$

$$2 \times 3 = 6$$

36 it is right.

2.) All the fish & people totals are all even.

3.) $\frac{3}{6}$ or half of the boat numbers are odd.

4.) $\frac{3}{6}$ or half of the boat numbers are even.

5.) A number sentence is $6 \times 2 \times 3 = 36$.

$$6 \text{ boats} \times 2 \text{ people} \times 3 \text{ fish} = 36 \text{ fish}$$

6.) There is 1 dozen of people in all.

7.) The boat sum is half of the people sum.

Expert Scoring Rationales

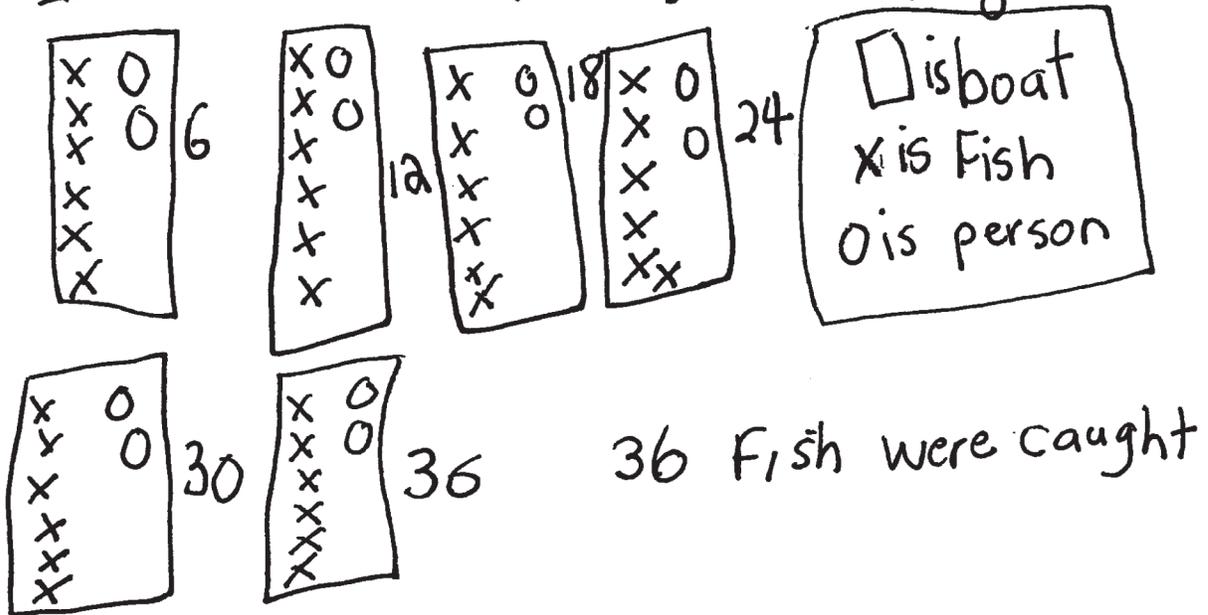
Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Expert</i>	<p>The student's strategy of using a diagram to determine the total number of fish caught by 12 people works to solve the task. The student's answer, "36 fish were caught," is correct. The student verifies her/his answer by making a table and equations. The student also verifies her/his answer by generalizing a rule for finding the total number of fish caught for any number of boats.</p>
Reasoning Proof <i>Expert</i>	<p>The student demonstrates correct reasoning of the underlying concepts in the problem. The student uses multiplication correctly. The student generalizes a rule, $6 \cdot b = f$, to verify that the data in her/his table is accurate, the answer is correct, and to find the number of fish for larger numbers of boats.</p>
Communication <i>Expert</i>	<p>The student correctly uses the mathematical terms <i>diagram</i>, <i>key</i>, <i>table</i>, <i>total</i>, <i>more than</i>, <i>dozen</i>, <i>pattern</i> and <i>rule</i>. The student correctly uses symbolic notation $6 \cdot b = f$ with the variables "b" and "f" defined in a key.</p>
Connections <i>Expert</i>	<p>The student makes the Practitioner mathematically relevant observations, "The boat pattern is + 1," "The friends pattern is + 2," "The fish pattern is + 6," "There are more total fish than boats or friends," "36 fish is 3 dozen." The student makes the Expert connection of generalizing a rule, $6 \cdot b = f$. The student supplies a key to define the variables, b and f, and states, "Each boat will always have two people in it." The student uses the rule to verify that the data in her/his table is accurate and states, "I am right because I get 36 fish all the time." The student also uses the rule to find the number of fish for 10 and 50 boats.</p>
Representation <i>Expert</i>	<p>The student's diagram is appropriate to the problem and accurate. A key is provided to define boat, fish, and person. The student's table is appropriate and accurate with all necessary labels and correctly entered data. The student uses her/his table to analyze the relationship between the boats and fish to generalize a rule to find the total number of fish for any number of boats.</p>

Expert

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

I need to find all the fish caught.

I will make a diagram. Key



Expert (cont.)

table

boat	Fishing friends	total fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

my connections
 The boat pattern is +1. The friends pattern is +2.
 The fish pattern is +6

There are more total fish than boats or friends
 36 fish is 3 dozen

I am right because I get 36 fish all the time.

key

b boat
 f fish

Each boat will always have two people in it

rule

$$6 \cdot b = f$$

$$6 \cdot 3 = 18$$

$$6 \cdot 6 = 36$$

$$6 \cdot 10 = 60$$

$$6 \cdot 50 = 300$$

I get 36 fish again.