

**New!**

**PROBLEM SOLVING**  
FOR THE **21<sup>ST</sup> CENTURY**  
BUILT FOR THE  
**COMMON CORE**

**A Classroom Resource Built for  
and Aligned to the Common Core**



500+ open-ended and engaging  
math tasks delivered online for teachers!

**K-5 Preview Material**

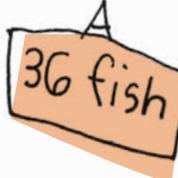
**We Set the Standards!**

I need to find out how many fish.  
I will make a diagram.

Key

○ boat			
☺ kid	6	12	18
fish			

			
24	30	36	

I saw the add 6 pattern  
I saw that there were 12 kids  
and that is a dozen

**Exemplars is the leader in performance-based assessment and instruction. We have been publishing problem-solving tasks for grades K–12 since 1993.**

***Problem Solving for the 21<sup>st</sup> Century: Built for the Common Core*** is a supplemental math resource for grades K–5 that features:

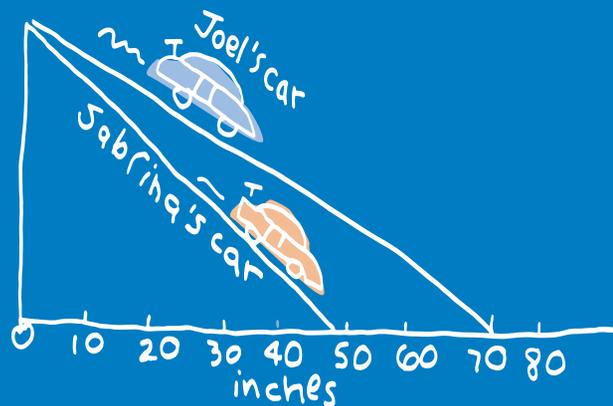
- Real-world problem solving uniquely developed for CCSSM.
- 500+ open-ended tasks that connect both the **Standards for Mathematical Content** and **Mathematical Practice**.
- Differentiated material for instruction, exploration and formative assessment.
- Summative assessments that include student anchor papers and scoring rationales.
- Standards-based rubrics that support the Mathematical Practices and provide teachers with clear guidelines for evaluating student work and providing meaningful feedback.
- Preliminary Planning Sheets that serve as the teacher's guide for every task, outlining the math concepts and skills that students will need to know as well as providing alternative strategies students may use to solve a problem.
- Online delivery for teachers through the Exemplars Library.

**Exemplars authentic tasks engage students and develop their abilities to reason and communicate mathematically and to formulate mathematical connections.**



“... School districts currently searching for reliable problems to enact the spirit and content of the Common Core at each grade will be delighted with this resource ... This can help teachers not only better understand the math of context-rich problems, but should also help them anticipate varied student outcomes for evaluation.”

Dr. H. Melnick  
Teacher & Leader Educator  
Bank Street College of Education  
New York



“... Exemplars has problems that allow for the constructivist approach demanded by the Common Core.”

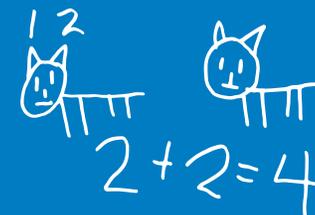
D. Personte  
New York

**exemplars.com**



“I am an elementary math coach whose job is to help teachers transition to the Core Curriculum. As we look at the Core and learn more, we see that students will need to write more and explain their thinking. Exemplars will help our students to begin writing more.”

K. Cooley  
Iowa



Rubrics are a valuable tool for assessing student work and an important component of Exemplars. Our assessment rubric allows teachers to examine student work against a set of analytic criteria.

### Standards-Based Math Rubric

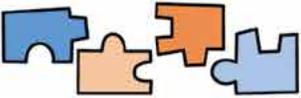
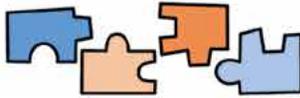
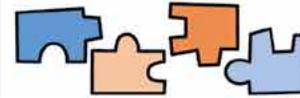
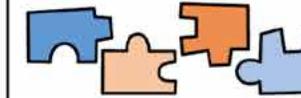
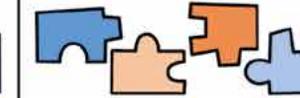
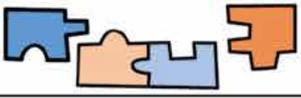
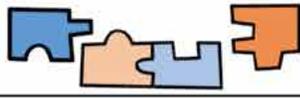
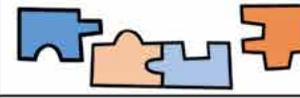
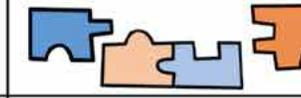
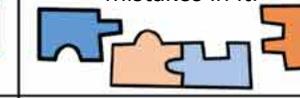
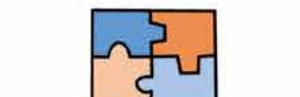
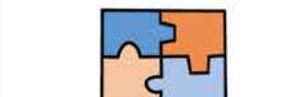
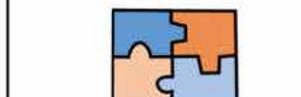
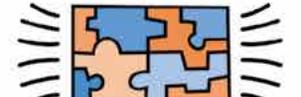
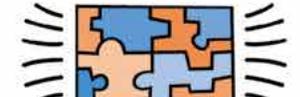
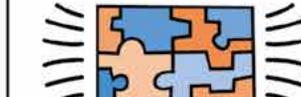
	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
<b>Novice</b>	<p>No strategy is chosen, or a strategy is chosen that will not lead to a solution.</p> <p>Little or no evidence of engagement in the task is present.</p>	<p>Arguments are made with no mathematical basis.</p> <p>No correct reasoning nor justification for reasoning is present.</p>	<p>No awareness of audience or purpose is communicated.</p> <p>No formal mathematical terms or symbolic notations are evident.</p>	<p>No connections are made or connections are mathematically or contextually irrelevant.</p>	<p>No attempt is made to construct a mathematical representation.</p>
<b>Apprentice</b>	<p>A partially correct strategy is chosen, or a correct strategy for solving only part of the task is chosen.</p> <p>Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.</p>	<p>Arguments are made with some mathematical basis.</p> <p>Some correct reasoning or justification for reasoning is present.</p>	<p>Some awareness of audience or purpose is communicated.</p> <p>Some communication of an approach is evident through verbal/written accounts and explanations.</p> <p>An attempt is made to use formal math language. One formal math term or one symbolic notation is evident.</p>	<p>A mathematical connection is attempted but is partially incorrect or lacks contextual relevance.</p>	<p>An attempt is made to construct a mathematical representation to record and communicate problem solving but is not accurate.</p>

There are four levels of performance (Novice, Apprentice, Practitioner and Expert). The Practitioner level is proficient and meets the standard. Our rubric criteria strongly support the Common Core Standards for Mathematical Practice. To learn more, refer to our crosswalk on pages 24–25.

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
<p><b>Practitioner</b></p> 	<p>A correct strategy is chosen based on mathematical situation in the task.</p> <p>Planning or monitoring of strategy is evident.</p> <p>Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</p> <p><i>Note: The Practitioner must achieve a correct answer.</i></p>	<p>Arguments are constructed with adequate mathematical basis.</p> <p>A systematic approach and/or justification of correct reasoning is present.</p>	<p>A sense of audience or purpose is communicated.</p> <p>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</p> <p>Formal math language is used to share and clarify ideas. At least two formal math terms or symbolic notations are evident in any combination.</p>	<p>A mathematical connection is made. Proper contexts are identified that link both the mathematics and the situation in the task.</p> <p>Some examples may include one or more of the following:</p> <ul style="list-style-type: none"> <li>• clarification of the mathematical or situational context of the task</li> <li>• exploration of mathematical phenomenon in the context of the broader topic in which the task is situated</li> <li>• noting patterns, structures and regularities</li> </ul>	<p>An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</p>
<p><b>Expert</b></p>	<p>An efficient strategy is chosen and progress toward a solution is evaluated.</p> <p>Adjustments in strategy, if necessary, are made along the way, and/or alternative strategies are considered.</p> <p>Evidence of analyzing the situation in mathematical terms and extending prior knowledge is present.</p> <p><i>Note: The Expert must achieve a correct answer.</i></p>	<p>Deductive arguments are used to justify decisions and may result in formal proofs.</p> <p>Evidence is used to justify and support decisions made and conclusions reached.</p>	<p>A sense of audience and purpose is communicated.</p> <p>Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.</p> <p>Formal math language is used to consolidate math thinking and to communicate ideas. At least one of the math terms or symbolic notations is beyond grade level.</p>	<p>Mathematical connections are used to extend the solution to other mathematics or to a deeper understanding of the mathematics in the task.</p> <p>Some examples may include one or more of the following:</p> <ul style="list-style-type: none"> <li>• testing and accepting or rejecting of a hypothesis or conjecture</li> <li>• explanation of phenomenon</li> <li>• generalizing and extending the solution to other cases</li> </ul>	<p>An appropriate mathematical representation is constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.</p>

Rubrics also present students with important information about what is expected and what kind of work meets the standard. Our student rubrics may be used to develop a child's ability to self- and peer-assess.

### Jigsaw Student Rubric

Level	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
<b>Novice</b> Makes an effort. No or little understanding.	I did not understand the problem. 	My math thinking is not correct. 	I used no math language and/or math notation. 	I did not notice anything about the problem or the numbers in my work. 	I did not use a math representation to help solve the problem and explain my work. 
<b>Apprentice</b> Okay, good try. Unclear if student understands.	I understand only part of the problem. My strategy works for part of the problem. 	Some of my math thinking is correct. 	I used some math language and/or math notation. 	I tried to notice something, but it is not about the math in the problem. 	I tried to use a math representation to help solve the problem and explain my work, but it has mistakes in it. 
<b>Practitioner</b> Excellent. Clear. Strong understanding. Meets the standard.	I understand the problem and my strategy works. My answer is correct. 	All of my math thinking is correct. 	I used math language and/or math notation accurately throughout my work. 	I noticed something about my math work. 	I made a math representation to help solve the problem and explain my work, and it is labeled and correct. 
<b>Expert</b> Wow, awesome! Exceptional understanding!	I understand the problem. My answer is correct. I used a rule, and/or verified that my strategy is correct. 	I showed that I knew more about a math idea that I used in my plan. Or, I explained my rule. 	I used a lot of specific math language and/or notation accurately throughout my work. 	I noticed something in my work, and used that to extend my answer and/or I showed how this problem is like another problem. 	I used another math representation to help solve the problem and explain my work in another way. 

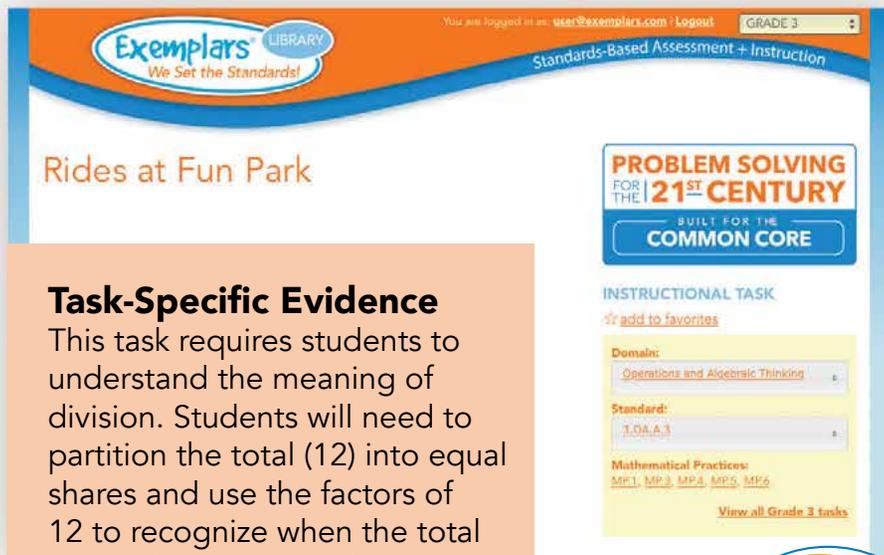
Through the Exemplars Library, both instructional tasks and corresponding summative assessments are provided online for each applicable Common Core Content Standard. Below is a sample of an instructional task. A summative assessment sample may be found on page 11.

## Instructional Task Sample: Grade 3

### Rides at Fun Park

#### Common Core Alignments:

- Content Standard: 3.OA.A.3
- Mathematical Practices: MP1, MP3, MP4, MP5, MP6



#### Task-Specific Evidence

This task requires students to understand the meaning of division. Students will need to partition the total (12) into equal shares and use the factors of 12 to recognize when the total cannot be partitioned into equal groups of five without having a remainder.

Material is delivered online for teachers!



### TASK

Twelve friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Roller Coaster has cars that hold six people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

To meet individual student's needs, each instructional task is differentiated to include a "more accessible" and a "more challenging" version. These problems may be used as opportunities for students to learn new mathematical strategies, language and representations. Instructional tasks may also be used in combination with Exemplars rubric for formative assessment.

## Alternative Versions of the Task

### More Accessible Version:

Twelve friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

### More Challenging Version:

Eighteen friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Roller Coaster has cars that hold six people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

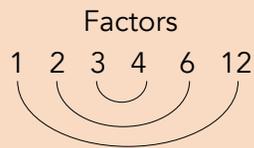
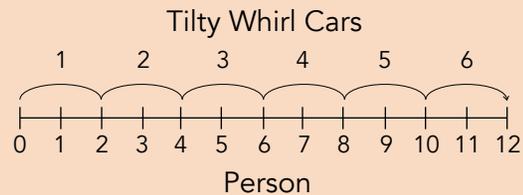


Possible solution representations and examples of mathematical connections are provided for each instructional task. Mathematical solutions are provided for the alternative versions.

## Possible Solutions

The 12 friends can ride the Tilty Whirl, Roller Coaster and the Ferris Wheel.

Ride	People per Car	Total Cars Filled	Remaining People
Tilty Whirl	2	6	0
Ferris Wheel	4	3	0
Roller Coaster	6	2	0
Water Log	5	2	2



$$2 \times 6 = 12$$

$$12 \div 4 = 3$$

$$12 - 6 = 6$$

$$6 - 6 = 0$$

$$5 + 5 = 10 \quad 2 \text{ remaining people}$$

## Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- 12 is a dozen friends.
- There is an even number of friends.
- There is an even number of friends in each car except for the Water Log, which holds an odd number.
- The Roller Coaster uses the least amount of cars.
- The Tilty Whirl uses the most cars.
- Multiplication is used to support addition.
- 1, 2, 3, 4, 6 and 12 are called factors of 12.
- Solve more than one way to verify the answer.
- Relate to another task and state a math link.

## Alternative Version Solutions

### More Accessible Version:

The 12 friends can ride the Tilty Whirl and the Ferris Wheel.

### More Challenging Version:

The 18 friends can ride the Tilty Whirl and the Roller Coaster.

Each instructional task is accompanied by a Preliminary Planning Sheet. Depending on the intended purpose of a task, this resource enables teachers to anticipate which math concepts and skills students might be required to use or what instruction should be given ahead of time.

## Rides at Fun Park

## Preliminary Planning Sheet

**Standard:** 3.OA.A.3  
**Math Practices:** MP1, MP3, MP4, MP5, MP6

### Major Underlying Mathematical Concepts

- Using multiplication/division within 100
- Partitioning into equal shares
- Factors of 12
- Finding a missing factor when the product and one factor are known
- Repeated subtraction of equal parts

### Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Array
- Chart
- Number line

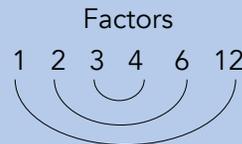
### Possible Mathematical Vocabulary/Symbolic Representation

- Model
- Diagram/Key
- Chart
- Number line
- Equivalent/Equal to
- Equal sets/Equal parts/Equal shares
- Sets/Groups
- Array
- Total/Sum
- Amount
- Most/Least
- Odd/Even
- Equation
- Quotient
- Divisor
- Dividend
- Product
- Factor
- Remainder
- Per
- Dozen

### Possible Solutions

The 12 friends can ride the Tilty Whirl, Roller Coaster and the Ferris Wheel.

Ride	People per Car	Total Cars Filled	Remaining People
Tilty Whirl	2	6	0
Ferris Wheel	4	3	0
Roller Coaster	6	2	0
Water Log	5	2	2



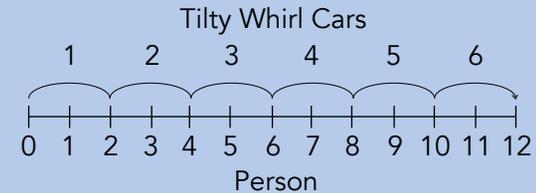
$$2 \times 6 = 12$$

$$12 \div 4 = 3$$

$$12 - 6 = 6$$

$$6 - 6 = 0$$

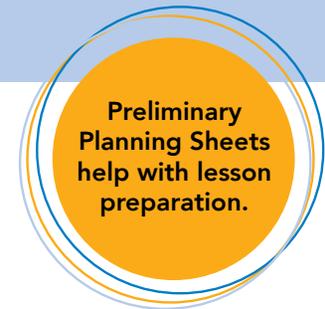
$$5 + 5 = 10 \quad 2 \text{ remaining people}$$



### Possible Connections

- 12 is a dozen friends.
- There is an even number of friends.
- The Tilty Whirl uses the most cars.
- The Roller Coaster uses the least amount of cars.
- There is an even number of friends in each car except for the Water Log, which holds an odd number.

- Multiplication is used to support addition.
- 1, 2, 3, 4, 6 and 12 are called factors of 12.
- Solve more than one way to verify the answer.
- Relate to another task and state a math link.



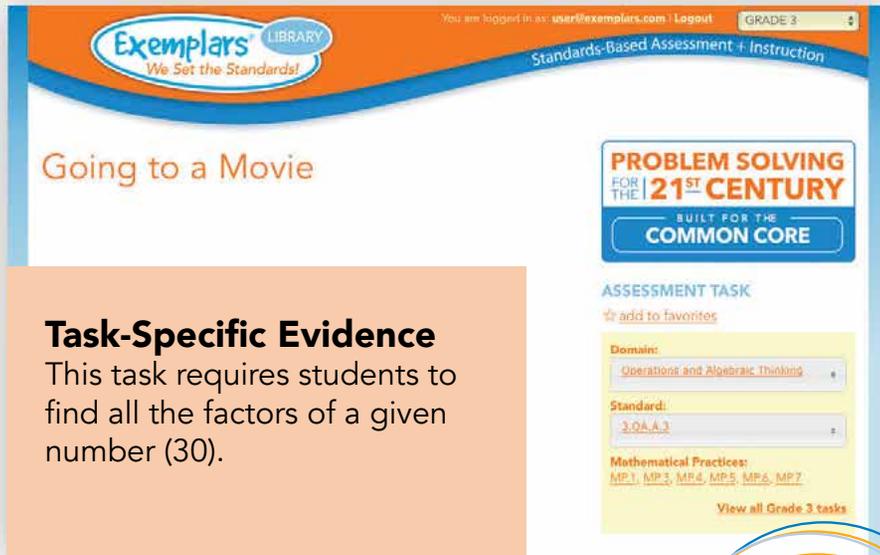
Summative assessments are provided for each applicable Common Core Content Standard. These tasks are designed to evaluate if a student meets the standard. Anchor papers and scoring rationales are included with each problem. These tasks are not differentiated.

## Summative Assessment Sample: Grade 3

### Going to a Movie

#### Common Core Alignments:

- Content Standard: 3.OA.A.3
- Mathematical Practices: MP1, MP3, MP4, MP5, MP6, MP7



**Task-Specific Evidence**  
This task requires students to find all the factors of a given number (30).

Additional task samples can be viewed online at [exemplars.com](http://exemplars.com)!



### TASK

Mr. Murphy is taking some students to see a movie. Mr. Murphy needs a seating arrangement for thirty students. Mr. Murphy wants an equal amount of students in each row. What are all the possible seating arrangements Mr. Murphy can make? Show all of your mathematical thinking.

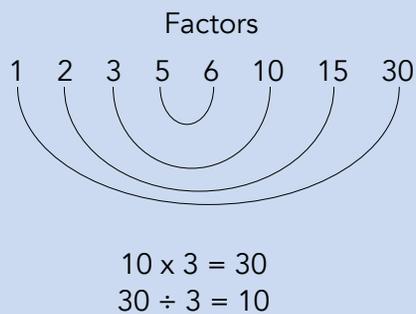
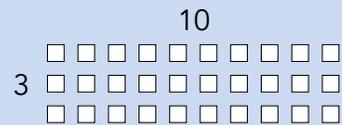
## Possible Solutions

The possible seating arrangements include:

1 x 30, 2 x 15, 3 x 10, 5 x 6, 6 x 5, 10 x 3, 15 x 2, and 30 x 1.

Rows	1	2	3	5	6	10	15	30
Seat	30	15	10	6	5	3	2	1

Key  
□ is seat



$30 \div 1 = 30$	30	
$30 \div 2 = 15$	<u>-6</u>	
$30 \div 3 = 10$	24	5 for 6
$30 \div 5 = 6$	<u>-6</u>	
$30 \div 6 = 5$	18	
$30 \div 10 = 3$	<u>-6</u>	
$30 \div 15 = 2$	12	
$30 \div 30 = 1$	<u>-6</u>	
	6	
	<u>-6</u>	
	0	

## Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- There are 8 factors of 30.
- 4 arrangements are “flipped” for the Commutative Property.
- There cannot be a remainder of seats or students, so the factors of 30 can be used to determine the seating arrangement.
- 2, 3, and 5 are prime numbers.
- Discuss equal versus even.
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.
- Discuss how area can be considered in a solution.
- Discuss why some arrangements would not be a good choice for the movies: 1 x 30, 2 x 15.
- Relate multiplication to division ( $5 \times 6 = 30$ ,  $30 \div 6 = 5$ ).

Each summative assessment task is accompanied by a Preliminary Planning Sheet (PPS). In this setting, the PPS allows teachers to foresee what instruction should be done before the summative assessment is given. It may also be used in combination with the Exemplars rubric to support teachers in assessing student work.

## Going to a Movie

## Preliminary Planning Sheet

Standard: 3.OA.A.3

Math Practices: MP1, MP3, MP4, MP5, MP6, MP7

### Major Underlying Mathematical Concepts

- Use multiplication/division within 100
- Combinations/Arrangement/Arrays
- Factors of 30

### Possible Problem-Solving Strategies

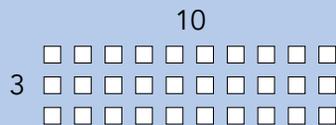
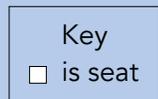
- Model (manipulatives)
- Diagram/Key
- Table
- Array

### Possible Mathematical Vocabulary/Symbolic Representation

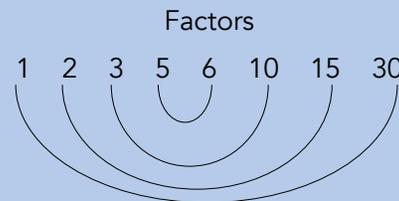
- Model
- Diagram/Key
- Table
- Array
- Product
- Factor
- Arrangements
- Combinations
- Odd/Even
- Pattern
- Equations
- Total/Sum
- Difference
- Equivalent/Equal to
- $3 \times 10$
- $2 \times 15$
- $1 \times 30$
- $5 \times 6$
- Commutative Property
- Set/Group
- Quotient
- Area
- Prime
- Row/Column

### Possible Solutions

The possible seating arrangements include:  
 $1 \times 30$ ,  $2 \times 15$ ,  $3 \times 10$ ,  $5 \times 6$ ,  $6 \times 5$ ,  $10 \times 3$ ,  $15 \times 2$ , and  $30 \times 1$ .



Rows	1	2	3	5	6	10	15	30
Seat	30	15	10	6	5	3	2	1



$10 \times 3 = 30$   
 $30 \div 3 = 10$

$30 \div 1 = 30$

$30 \div 2 = 15$

$30 \div 3 = 10$

$30 \div 5 = 6$

$30 \div 6 = 5$

$30 \div 10 = 3$

$30 \div 15 = 2$

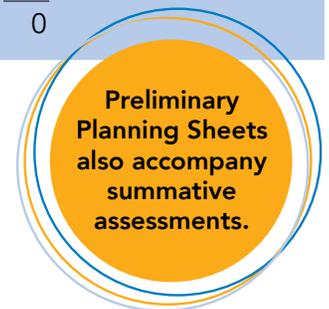
$30 \div 30 = 1$

$30$   
 $- 6$   
 $24$  5 for 6  
 $- 6$   
 $18$   
 $- 6$   
 $12$   
 $- 6$   
 $6$   
 $- 6$   
 $0$

### Possible Connections

- There are 8 factors of 30.
- 4 arrangements are “flipped” for the Commutative Property.
- Discuss why some arrangements would not be a good choice for the movies:  $1 \times 30$ ,  $2 \times 15$ .
- There cannot be a remainder of seats or students, so the factors of 30 can be used to determine the seating arrangement.

- 2, 3, and 5 are prime numbers.
- Discuss equal versus even.
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.
- Discuss how area can be considered in a solution.
- Relate multiplication to division ( $5 \times 6 = 30$ ,  $30 \div 6 = 5$ ).

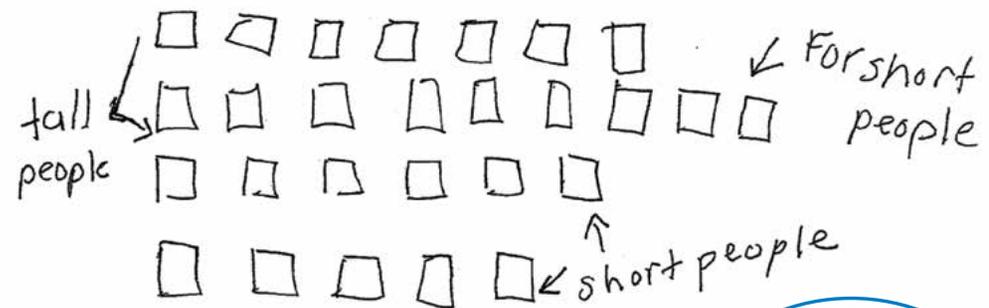


## NOVICE

P/S	R/P	Com	Con	Rep	A/Level
N	N	A	N	A	N

Criteria and Performance Levels	Scoring Rationales
<b>Problem Solving</b> <i>Novice</i>	The student's strategy of diagramming seats in four unequal rows and stating which seats would suit students of various heights would not lead to a correct solution. The student's answer, "Do it this way," is incorrect.
<b>Reasoning &amp; Proof</b> <i>Novice</i>	No correct reasoning of the underlying mathematical concepts of using multiplication and division within 100 involving equal groups, arrays, and measurement quantities is evident. The student does not demonstrate any understanding of arrangements of equal groups, arrays, or factors of 30.
<b>Communication</b> <i>Apprentice</i>	The student correctly uses the mathematical term <i>diagram</i> and indicates understanding of the term but not the concepts involved in the task.
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
<b>Representation</b> <i>Apprentice</i>	The student's diagram represents seats, but the purpose of the seats representing students of various heights is not appropriate to the task.

I Have to see how the kids shuld sit. I will do a diagram



Do it this way.

Scoring rationales describe why each piece of student work is assessed at a specific performance level for each of the Exemplars rubric criterion.

The Apprentice level has the broadest range, from a student who is just beginning to demonstrate mathematical understanding to a student who almost meets the standard.

## APPRENTICE Student 1

Criteria and Performance Levels	Scoring Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student's strategy of using a diagram to make arrangements of how to seat 30 students would work to solve the task. The student's solution is correct for only part of the task. The student identifies only six of the eight possible seating arrangements.
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student demonstrates correct reasoning of the underlying mathematical concepts of using multiplication and division within 100 involving equal groups, arrays, and measurement quantities that is necessary to solve the task.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical term <i>arrangement</i> from the task. The student also correctly uses the terms <i>diagram</i> and <i>factors</i> . The student correctly uses the mathematical notation $10 \times 3$ (10 by 3), $3 \times 10$ , $6 \times 5$ , $5 \times 6$ , $1 \times 30$ , $30 \times 1$ .
<b>Connections</b> <i>Practitioner</i>	The student makes a mathematically relevant observation about her/his solution, "They are all factors of 30—5, 6, 10, 3, 1, 30." The student does not need to list all the factors of 30.
<b>Representation</b> <i>Apprentice</i>	The student's diagram is appropriate but not accurate. There is no key or text to indicate that the arrays represent student seats. The $2 \times 15$ and $15 \times 2$ arrays are missing.

P/S	R/P	Com	Con	Rep	A/Level
A	P	P	P	A	A

I need to find out how many seating arrangements I can make. I will make a diagram.

30  
10000000000000000000000000000000

6  
000000  
000000  
000000  
000000  
000000

5  
0000  
0000  
0000  
0000  
0000

3  
000  
000  
000  
000  
000

10  
0000000000  
3000000000  
0000000000

30  
00000000000000000000000000000000

Connection  
They are all factors of 30  
5, 6, 10,  
3, 1, 30

10x3 = 30  
3x10 = 30  
6x5 = 30  
5x6 = 30  
1x30 = 30  
30x1 = 30

Answer  
10x3 6x5  
3x10 5x6  
1x30  
30x1

Throughout this resource there is often more than one anchor paper associated with a level of performance. These are intended to demonstrate different strategies a student might use or different misconceptions a student might have (depending on the performance level).

## APPRENTICE Student 2 (continued on next page)

Criteria and Performance Levels	Scoring Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student's strategy of making a diagram and key of one possible seating arrangement for 30 students and then using grid paper to diagram that arrangement and five others would work to solve the task. The student does not diagram all the possible arrangements. The 2 x 15 and 15 x 2 arrangements are missing.
<b>Reasoning &amp; Proof</b> <i>Apprentice</i>	The student's solution contains some understanding of the underlying mathematical concepts of equal groups/arrays, but there are gaps in how the student solved the task and the reasoning used. It is not clear why the student draws six arrays and states an answer of 10. Two arrangements/arrays are labeled 1 x 2, which is not consistent with the others.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical term <i>arrangement</i> from the task. The student also correctly uses the mathematical term <i>key</i> .
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
<b>Representation</b> <i>Apprentice</i>	The student's use of two different types of diagrams to show arrangements of student seating is appropriate to this task. The student's first diagram is accurate for one possible arrangement. There is a key to define the "kids" seating in each row and column. The student's arrangements/arrays on the grid paper all lack labels or a key to indicate what the arrays are indicating. The two 1 x 2 diagrams should read 1 x 30 and 30 x 1.

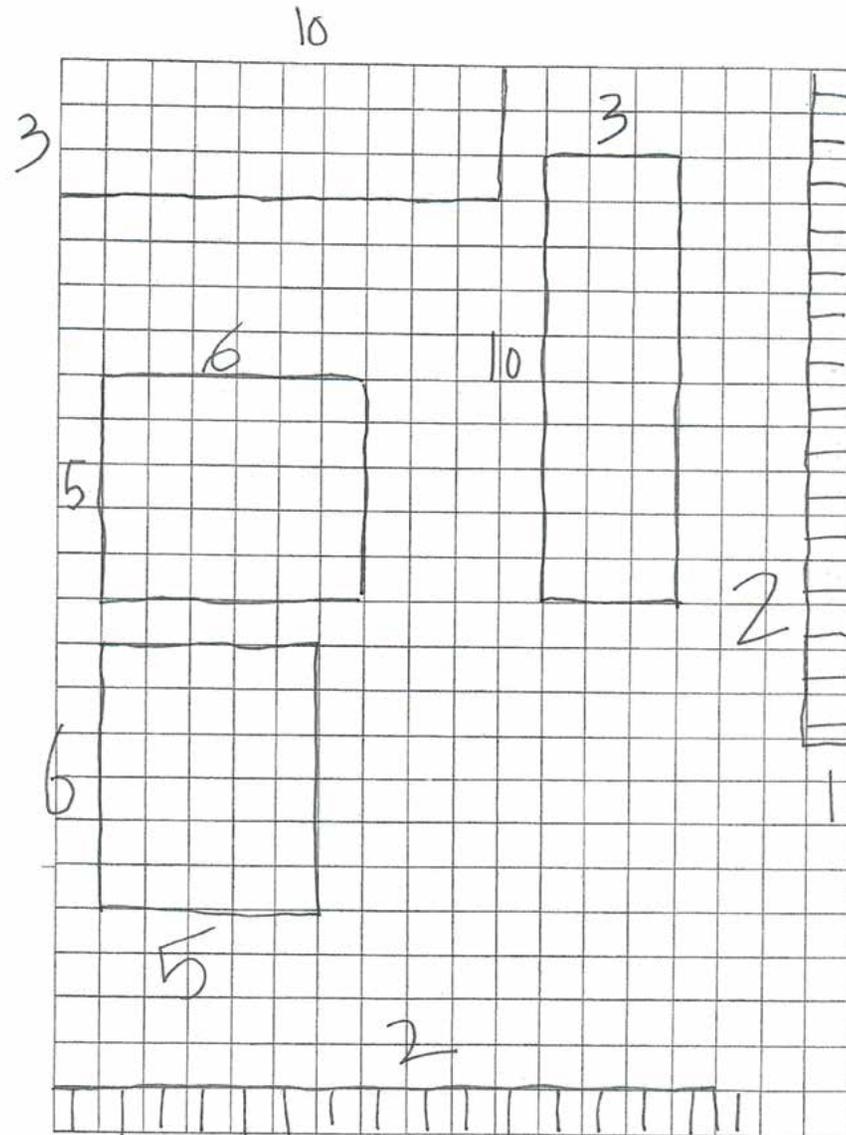
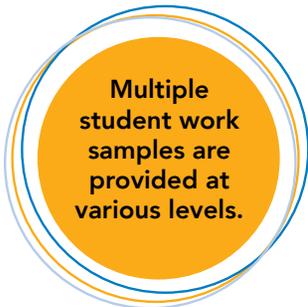
What are all the seating arrangements Mr. Murphy can make.  
I know he is taking his class to the movie. I know there are thirty students.

key  
numbers is 1 Kid

10

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30

## APPRENTICE Student 2



P/S	R/P	Com	Con	Rep	A/Level
A	A	P	N	A	A

## PRACTITIONER (continued on next page)

Criteria and Performance Levels	Scoring Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of making a diagram of arrays to find eight seating arrangements works for solving this task. The problem asks, "What are all the seating arrangements Mr. Murphy can make?" The student correctly diagrams eight arrays/arrangements and is not required to state $5 \times$ , etc. <i>Note: Students can just indicate four possible arrangements for a correct answer as the understanding of Commutative Property can be assumed.</i>
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student demonstrates correct reasoning of the underlying mathematical concepts of arrangements/arrays.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>arrangement</i> , <i>equal</i> , <i>amount</i> , <i>row</i> from the task. The student also correctly uses the terms <i>array</i> and <i>key</i> . The student does not use the term "multipuls" (multiples) correctly so it cannot be considered as a mathematical term for this task.
<b>Connections</b> <i>Practitioner</i>	The student connects her/his arrays to the factors of 30 in two ways. The student shows 1 with 30, 2 with 15, 3 with 10, and 5 with 6. The student also lists the multiplication notation with the reversals $3 \times 10 = 30$ , $10 \times 3 = 30$ ... The student does not earn credit for stating, "multipuls" 30, 60, 80, 100, because this is not correct.
<b>Representation</b> <i>Practitioner</i>	The student's arrays are appropriate and accurate for this task. A key is provided to define what the arrangements/arrays are representing.

What are all the seating arrangements Mr. Murphy can make?  
 I know that Mr. Murphy is trying to seat his children equal amount in each row

Arrays      Key  
 $\square = 1 \text{ seat}$

times =  $\times$   
 $3 \times 10 = 30$   
 $10 \times 3 = 30$   
 $5 \times 6 = 30$   
 $6 \times 5 = 30$   
 $2 \times 15 = 30$   
 $15 \times 2 = 30$   
 $1 \times 30 = 30$   
 $30 \times 1 = 30$

Multipuls  
 30, 60, 80, 100

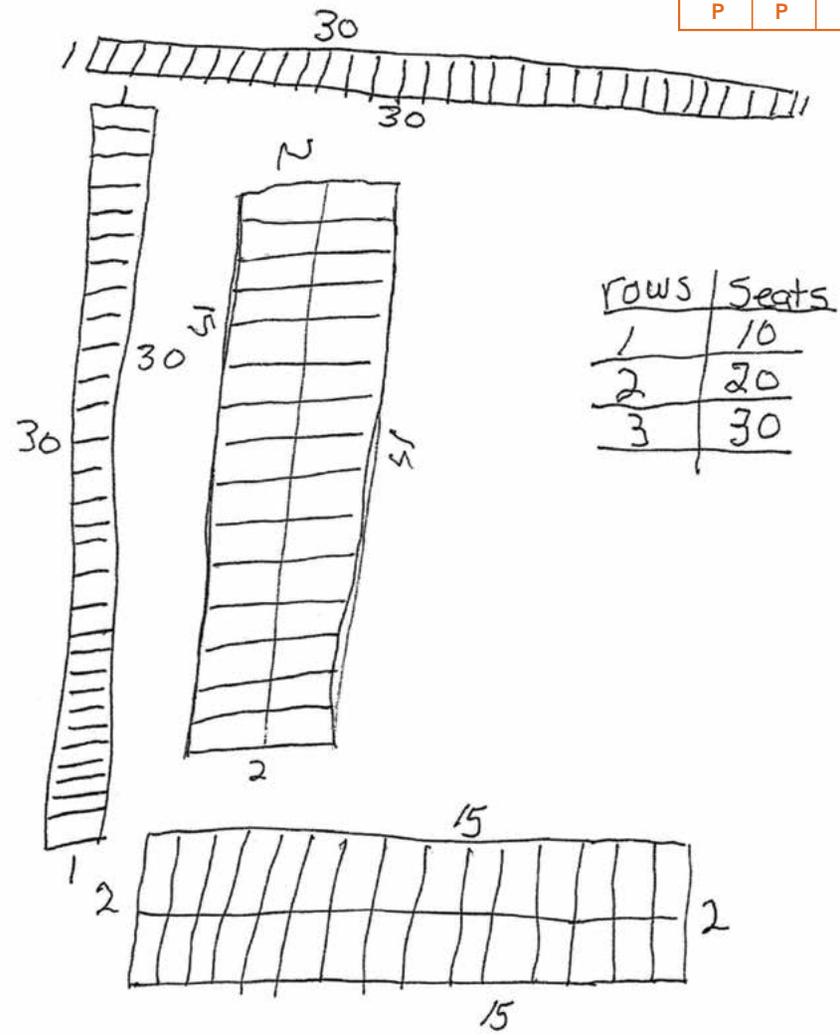
# PRACTITIONER



The Goal!

This student meets the standard.

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P



**EXPERT** (continued on next page)

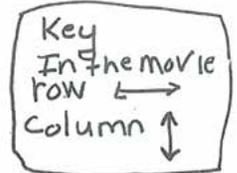
Criteria and Performance Levels	Scoring Rationales
<b>Problem Solving</b> <i>Expert</i>	The student's strategy of making a table to find the eight possible arrangements/arrays for 30 people works for solving the task. The student's answer, "8 arrangements," is correct. The student also extends her/his work to the concept of area.
<b>Reasoning &amp; Proof</b> <i>Expert</i>	The student demonstrates correct understanding of the underlying mathematical concept of arrangements/arrays. The student brings correct reasoning of the the underlying concept of area to extend her/his solution.
<b>Communication</b> <i>Expert</i>	The student correctly uses the mathematical terms <i>arrangement</i> and <i>row</i> from the task. The student also correctly uses the terms <i>table</i> , <i>column</i> , <i>key</i> , <i>factors</i> , <i>commutative property</i> , <i>array</i> , <i>square</i> and <i>area</i> . The student uses the mathematical notation $1 \times 30$ (1 by 30), $30 \times 1$ , $2 \times 15$ , $15 \times 2$ , $3 \times 10$ , $10 \times 3$ , $5 \times 6$ , $6 \times 5$ .
<b>Connections</b> <i>Expert</i>	The student states, "I can see factors on my table for 30," and lists them on her/his paper. The student then states, "the commutative property." The student analyzes the relationships among elements in her/his solution. The student states, "If you think about it the $1 \times 30$ and $30 \times 1$ would be silly in the movies. Best is $5 \times 6$ or $6 \times 5$ , you can be together more. I will show you." The student uses arrangements/arrays to link the concept of area in a $6 \times 5$ arrangement/array as opposed to the area in a $1 \times 30$ arrangement/array. The student states, "This is better area for a movie. 30 squares of area used good. This area is not good for 30 squares." The student also defines the use of area and direction by indicating that the $1 \times 30$ would have seats "to far to sit" and "to close to sit."
<b>Representation</b> <i>Expert</i>	The student's table is appropriate and accurate. All necessary labels are provided and the entered data is correct. The student's diagram is also appropriate and accurate to support her/his understanding of area.

I have to find when Mr. Murphy has all his arrangements. I will make a table.

**MY WORK**

Seating Arrangements

	row	column	
$30 \div 1 = 30$	1	30	$1 \times 30 = 30$
$30 \div 2 = 15$	2	15	$2 \times 15 = 30$
$30 \div 3 = 10$	3	10	$3 \times 10 = 30$
$30 \div 5 = 6$	5	6	$5 \times 6 = 30$
$30 \div 6 = 5$	6	5	$6 \times 5 = 30$
$30 \div 10 = 3$	10	3	$10 \times 3 = 30$
$30 \div 15 = 2$	15	2	$15 \times 2 = 30$
$30 \div 30 = 1$	30	1	$30 \times 1 = 30$



**ANSWER**

8 arrangements

I can see factors on my table for 30,  
 $1 \times 30$  is  $30 \times 1$   
 $2 \times 15$  is  $15 \times 2$

the commutative property  
 $3 \times 10$  is  $10 \times 3$   
 $5 \times 6$  is  $6 \times 5$

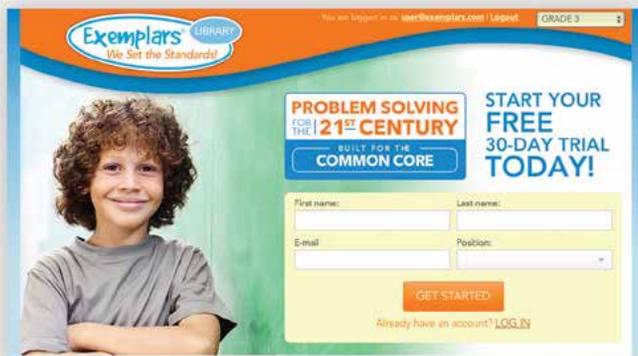
**MY CONNECTIONS**

if you think about it the  $1 \times 30$  and  $30 \times 1$  would be silly in the movies. Best is  $5 \times 6$  or  $6 \times 5$  you can be

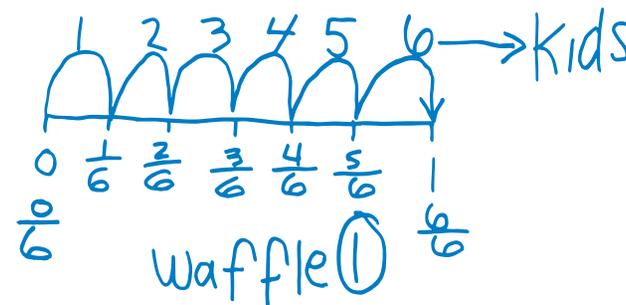


## Sign up for a FREE 30-day Trial!

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# Exemplars Advantages

## For Teachers:

- Authentic performance material to meet the Common Core Standards for Mathematical Content and Mathematical Practice.
- Differentiated versions to use with students at varying levels.
- Standards-based scoring rubrics at four levels of performance.
- Preliminary Planning Sheets that serve as a useful teaching resource.
- Anchor papers and scoring rationales that provide guidelines for assessment and offer concrete examples of student work at each level of the rubric.
- Tools to develop students' critical thinking and reasoning skills as well as their abilities to self- and peer-assess.
- A method of formative assessment.

## For Students:

- Real-world tasks that captivate student's interest and develop their critical thinking, reasoning and communication skills.
- Student rubrics, anchor papers and scoring rationales that provide a basis for peer- and self-assessment.
- Differentiated material to meet the needs of each student.

## For Curriculum Leaders and Staff Developers:

- Open-ended performance tasks to help meet the Common Core standards.
- Material that can be used to introduce teachers to problem solving.
- Rubrics and anchor papers provide consistency for working with teachers on scoring performance-assessment problems.
- Material provides a basis for reviewing student work and tying together assessment results, curriculum and instruction.



Below is a crosswalk showing the correlation between Exemplars assessment rubric and the Common Core Standards for Mathematical Practice.

The CCSS for Mathematical Practice are comprised of:	Exemplars rubric criteria from the "Practitioner Level" supports CCSSM by requiring students to do the following in order to meet the standard:
<p><b>1) Make sense of problems and persevere in solving them.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>• A correct strategy is chosen based on mathematical situation in the task.</li> <li>• Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</li> <li>• Planning or monitoring of a strategy is evident.</li> </ul> <p><b>Reasoning and Proof</b></p> <ul style="list-style-type: none"> <li>• A systematic approach and/or justification of correct reasoning is present.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>• Clarification is made of the mathematical or situational context of the task.</li> <li>• An exploration of mathematical phenomenon is made in the context of the broader topic in which the task is situated.</li> </ul> <p><b>Representation</b></p> <ul style="list-style-type: none"> <li>• An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</li> </ul>
<p><b>2) Reason abstractly and quantitatively.</b></p>	<p><b>Reasoning and Proof</b></p> <ul style="list-style-type: none"> <li>• Arguments are constructed with adequate mathematical basis.</li> <li>• A systematic approach and/or justification of correct reasoning is present.</li> </ul> <p><b>Representation</b></p> <ul style="list-style-type: none"> <li>• An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>• Clarification is made of the mathematical or situational context of the task.</li> <li>• An exploration of mathematical phenomenon is made in the context of the broader topic in which the task is situated.</li> </ul> <p><b>Communication</b></p> <ul style="list-style-type: none"> <li>• Formal math language is used to share and clarify ideas.</li> </ul>
<p><b>3) Construct viable arguments and critique the reasoning of others.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>• Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</li> </ul> <p><b>Reasoning and Proof</b></p> <ul style="list-style-type: none"> <li>• Arguments are constructed with adequate mathematical basis.</li> <li>• A systematic approach and/or justification of correct reasoning are/is present.</li> </ul> <p><b>Communications</b></p> <ul style="list-style-type: none"> <li>• A sense of audience or purpose is communicated.</li> <li>• Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>• Clarification is made of the mathematical or situational context of the task.</li> <li>• An exploration of mathematical phenomenon is made in the context of the broader topic in which the task is situated.</li> </ul> <p><b>Representation</b></p> <ul style="list-style-type: none"> <li>• An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</li> </ul>

Exemplars performance material provides teachers with a tool to use with their students to elicit the Mathematical Practices.

The CCSS for Mathematical Practice are comprised of:	Exemplars rubric criteria from the "Practitioner Level" supports CCSSM by requiring students to do the following in order to meet the standard:
<p><b>4) Model with mathematics.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</li> <li>Planning or monitoring of strategy is evident.</li> </ul> <p><b>Reasoning and Proof</b></p> <ul style="list-style-type: none"> <li>Arguments are constructed with adequate mathematical basis.</li> <li>A systematic approach and/or justification of correct reasoning are/is present.</li> </ul> <p><b>Representation</b></p> <ul style="list-style-type: none"> <li>An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>Clarification is made of the mathematical or situational context of the task.</li> <li>An exploration of mathematical phenomenon is made in the context of the broader topic in which the task is situated.</li> </ul> <p><b>Communication</b></p> <ul style="list-style-type: none"> <li>Formal math language is used throughout the solution to share and clarify ideas.</li> </ul>
<p><b>5) Use appropriate tools strategically.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>A correct strategy is chosen based on the mathematical situation in the task.</li> <li>Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</li> <li>Planning or monitoring of strategy is evident.</li> </ul>
<p><b>6) Attend to precision.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>The Practitioner must achieve a correct answer.</li> </ul> <p><b>Representation</b></p> <ul style="list-style-type: none"> <li>An appropriate and accurate mathematical representation is constructed and refined to solve problems or portray solutions.</li> </ul> <p><b>Communication</b></p> <ul style="list-style-type: none"> <li>A sense of audience or purpose is communicated.</li> <li>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</li> <li>Formal math language is used throughout the solution to share and clarify ideas.</li> <li>At least two formal math terms or symbolic notations are evident, in any combination.</li> </ul>
<p><b>7) Look for and make use of structure.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>Planning or monitoring of strategy is evident.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>Clarification is made of the mathematical or situational context of the task.</li> <li>An exploration of mathematical phenomenon is made in the context of the broader topic in which the task is situated.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>A mathematical connection is made. Proper contexts are identified that link both the mathematics in the situation and the task.</li> </ul>
<p><b>8) Look for and express regularity in repeated reasoning.</b></p>	<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>Planning or monitoring of strategy is evident.</li> </ul> <p><b>Connections</b></p> <ul style="list-style-type: none"> <li>A mathematical connection is made.</li> <li>Patterns, structures and regularities are noted.</li> </ul>

## Take your teachers' Common Core readiness to a whole new level with Exemplars professional development!

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“The presenter was so interesting, inspiring and encouraging – helped me feel as though I could truly do this in the classroom. My students will really benefit from this approach to learning.”

K. Dietz  
Kindergarten Teacher



## Common Core Math Sessions, K–12

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- Standards-Based Classroom Modeling: Problem Solving and Formative Assessment for the Common Core
- Assessing Student Performance Against the Common Core and Using Results to Improve Performance
- Integrating Content, Problem Solving and Assessment in the Classroom to Support the Common Core Standards for Mathematical Content and Mathematical Practice

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